Systemtheorie
Übungsaufgaben 3
Lösungen

BTI4

Bachelor-Studiengang Telekommunikation und Informationstechnik

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Diskrete Systeme

Aufgabe 1:

- Nichtlinear
- Nicht kausal
- Nicht stabil
- Zeitvariant

Aufgabe 2:

$$H(z) = \frac{3z^2}{z^2 - 0.25z - 0.125} = \frac{3z^2}{(z-0.5)(z+0.25)}$$

$$z_1 = 0.5$$

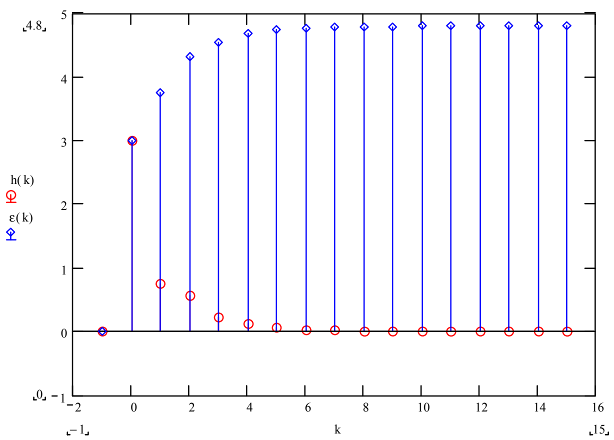
$$z_2 = -0.25$$

$$H(z) = 3 - \frac{0.25}{z+0.25} + \frac{1}{z-0.5}$$

$$h[k] = (-0.25)^k \varepsilon[k] + 2 \cdot 0.5^k \varepsilon[k]$$

$$\mathrm{E}(z) = \frac{3z^3}{(z+0.25)(z-0.5)(z-1)}$$

$$\varepsilon[k] = 4.8 - 2 \cdot 0.5^k \varepsilon[k] + 0.2 \cdot (-0.25)^k \varepsilon[k]$$



Diskrete Systeme

Aufgabe 3:

$$y[k] = C_1(0.5)^k + C_2(-0.25)^k$$

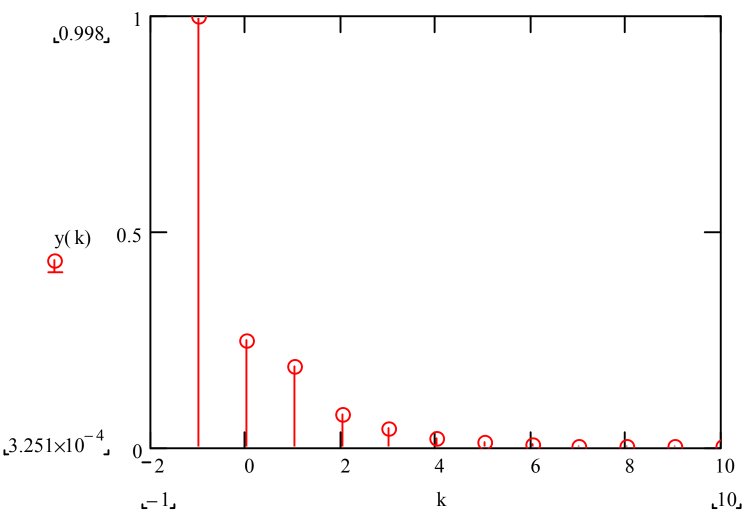
$$C_1(0.5)^{-1} + C_2(-0.25)^{-1} = 1$$

$$C_1(0.5)^0 + C_2(-0.25)^0 = 0.25$$

$$C_1 = 0.333$$

$$C_2 = -0.083$$

$$y[k] = 0.333(0.5)^k \varepsilon[k] - 0.083(-0.25)^k \varepsilon[k]$$



Diskrete Systeme

Aufgabe 4:

a) Das Filter ist ein FIR Filter. System ist stabil.

$$h[k] = \delta[k] - \delta[k-8]$$

$$\sum_{k=-\infty}^{\infty} \left| h[k] \right| = 2$$

b) Das Filter ist ein IIR Filter. System ist stabil.

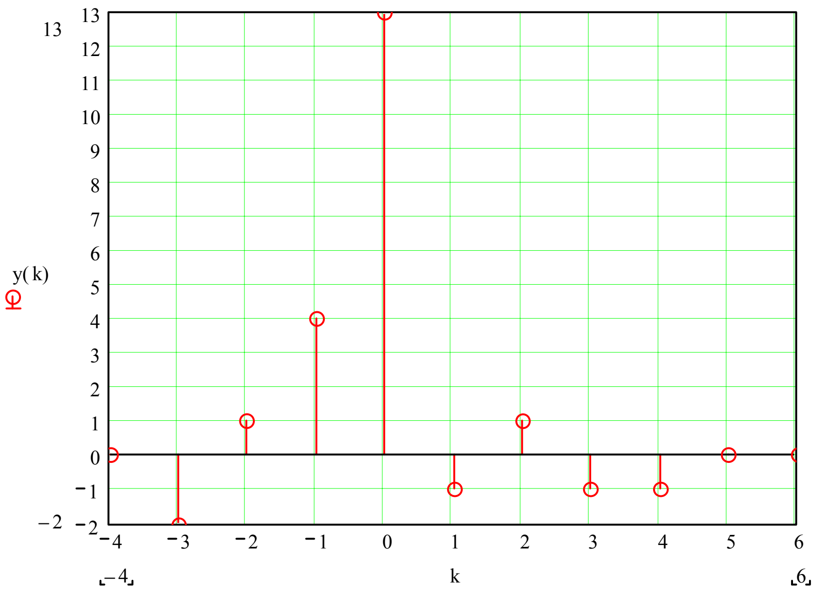
$$H(z) = \frac{2z}{z+0.9}$$

$$h[k] = 2(-0.9)^k \varepsilon[k]$$

$$\sum_{k=-\infty}^{\infty} \left| h[k] \right| = 2\sum_{n=0}^{\infty} \left| (-0.9)^k \right| = 2 \cdot 10$$

Diskrete Systeme

Aufgabe 5:



Diskrete Systeme

Aufgabe 6:

$$H\left(e^{j\Omega}\right) = \sum_{k=-\infty}^{\infty} h[k] e^{-jk\Omega} = 1 - e^{-j10\Omega}$$

$$x[k] = \frac{1}{2}\left(e^{-jk\frac{\pi}{10}} + e^{jk\frac{\pi}{10}} + e^{-jk\frac{\pi}{5}} + e^{jk\frac{\pi}{5}}\right)$$

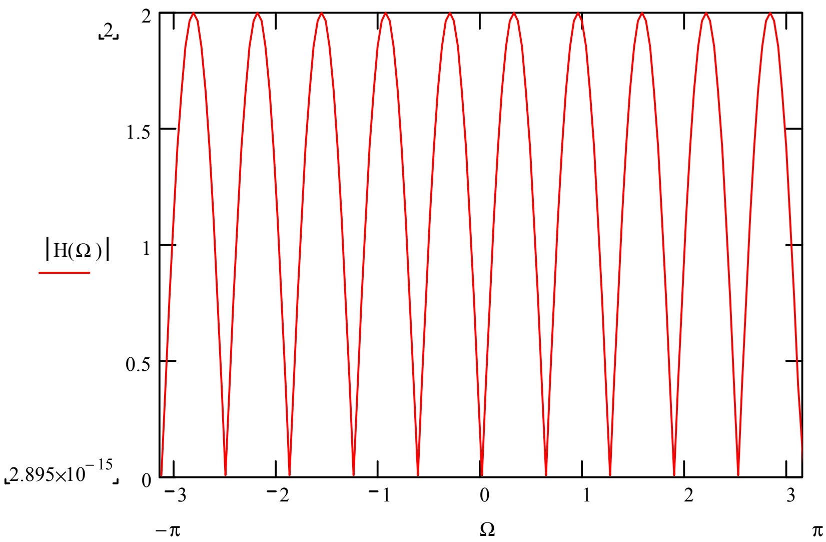
$$X\left(e^{j\Omega}\right) = 2\pi \cdot \delta\left(\Omega - \frac{\pi}{10}\right) + 2\pi \cdot \delta\left(\Omega + \frac{\pi}{10}\right) + 2\pi \cdot \delta\left(\Omega - \frac{\pi}{5}\right) + 2\pi \cdot \delta\left(\Omega + \frac{\pi}{5}\right)$$

$$H\left(e^{j\frac{\pi}{10}}\right) = 2$$

$$H\left(e^{j\frac{\pi}{5}}\right) = 0$$

$$Y\left(e^{j\Omega}\right) = 4\pi \cdot \delta\left(\Omega - \frac{\pi}{10}\right) + 4\pi \cdot \delta\left(\Omega + \frac{\pi}{10}\right)$$

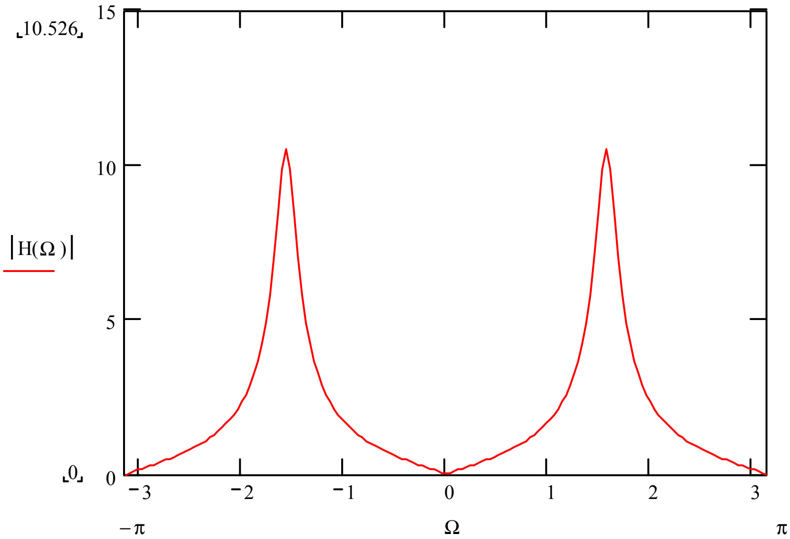
$$y[k] = 2\cos\left(k\frac{\pi}{10}\right)$$



Aufgabe 7:

a)

$$H\left(e^{j\Omega}\right)=\frac{1-e^{-j2\Omega}}{1+0.81e^{-j2\Omega}}$$



b)

$$H\left(e^{j0}\right)=0$$

$$H\left(e^{j\frac{\pi}{2}}\right)=10.53$$

$$H\left(e^{j\pi}\right)=0$$

$$y[k]=105.3\cos\left(k\frac{\pi}{2}\right)$$

Z-Transformation

Aufgabe 1:

$$A\varepsilon[k] \circ\!\!-\!\!\bullet \frac{Az}{z - 1} \qquad |z| > 1$$

$$x[k] = A\cos(k\Omega)\varepsilon[k] = \frac{1}{2}A\left(e^{jk\Omega} + e^{-jk\Omega}\right)\varepsilon[k]$$

$$X(z) = \frac{1}{2}A\left(\frac{z}{z - e^{j\Omega}} + \frac{z}{z - e^{-j\Omega}}\right) = A\left(\frac{z(z - \cos\Omega)}{z^2 - 2z\cos\Omega + 1}\right), |z| > 1$$

Aufgabe 2:

$$H(z) = \frac{z^5 + 1}{z^5 - 4z^4 + 6z^3 - 4z^2}$$

Nullstellen:

$$z^5 = -1$$

$$z_1 = e^{j\frac{\pi}{5}}, z_2 = e^{j\frac{3\pi}{5}}, z_3 = e^{j\frac{5\pi}{5}}, z_4 = e^{j\frac{7\pi}{5}}, z_5 = e^{j\frac{9\pi}{5}}$$

Polstellen:

$$z^2\left(z^3 - 4z^2 + 6z - 4\right) = 0$$

$$z^2\left(z^2 - 2z + 2\right)(z - 2) = 0$$

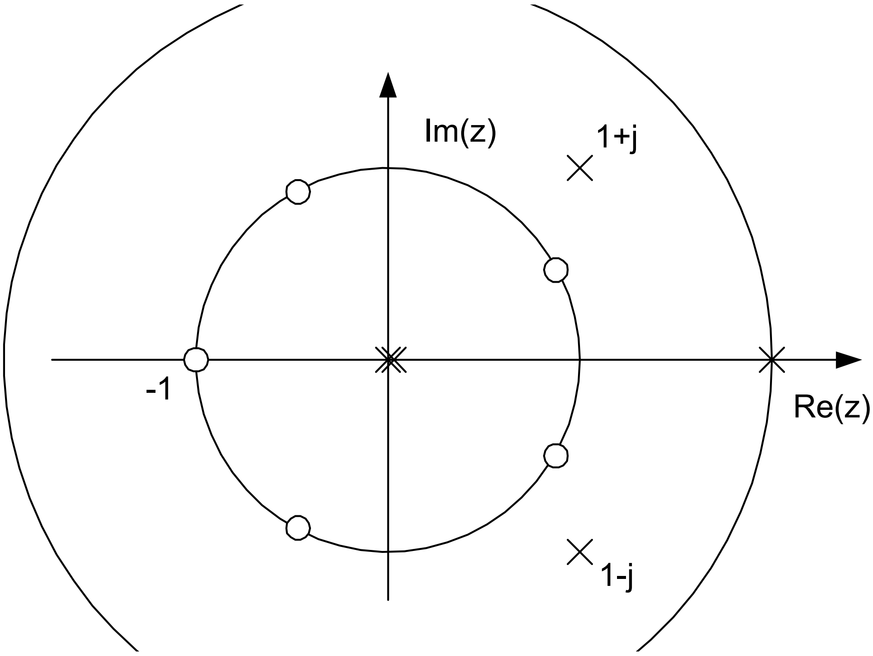
$$z_{1,2} = 0$$

$$z_3 = 1 + j$$

$$z_4 = 1 - j$$

$$z_5 = 2$$

Z- Transformation



Das System ist nicht stabil.

Z- Transformation

Aufgabe 3:

a)

Polstellen:

$$z_1 = 1$$

$$z_2 = 2$$

$$Y(z) = \frac{6z^2 - 10z + 2}{(z-1)(z-2)}$$

$$\frac{Y(z)}{z} = \frac{6z^2 - 10z + 2}{z(z-1)(z-2)} = \frac{1}{z} + \frac{2}{(z-1)} + \frac{3}{(z-2)}$$

b)

i)

$$y[k] = \delta[k] + \left(2 \cdot 1^k + 3 \cdot 2^k\right)\varepsilon[k]$$

ii)

$$y[k] = \delta[k] - \left(2 \cdot 1^k + 3 \cdot 2^k\right)\varepsilon[-k-1]$$

iii)

$$y[k] = \delta[k] + 2 \cdot 1^k \varepsilon[k] - 3 \cdot 2^k \varepsilon[-k-1]$$

z-Transformation

Aufgabe 4:

Einseitige z-Transformation:

$$G(z) = \sum_{k=0}^{\infty} g[k] z^{-k}$$

$$\begin{aligned}
Z\left[g[k-m]\right] &= \sum_{k=0}^{\infty} g[k-m] z^{-k} \\
&= \sum_{r=-m}^{\infty} g[r] z^{-(r+m)} \\
&= \sum_{r=-m}^{-1} g[r] z^{-(r+m)} + \sum_{r=0}^{\infty} g[r] z^{-r} z^{-m} \\
&= \sum_{r=-m}^{-1} g[r] z^{-(r+m)} + z^{-m} G(z)
\end{aligned}$$

Transformation der Differenzengleichung:

$$\begin{aligned}
Y(z) + 0.3\left(z^{-1}Y(z) + z^{-0}y(-1)\right) - 0.4\left(z^{-2}Y(z) + z^{-1}y(-1) + z^{-0}y(-2)\right) \\
= 0.5\left(X(z) + z^{-1}X(z) + z^{-0}x(-1)\right)
\end{aligned}$$

$$X(z) = \frac{z}{z-1}$$

$$x(-1) = 0$$

$$Y(z) = \frac{0.6z^3 + 0.8z^2 - 0.4z}{\left(z^2 + 0.3z - 0.4\right)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{-0.28}{z+0.8} + \frac{-0.231}{z-0.5} + \frac{1.111}{z-1}$$

$$y[k] = \left(-0.28(-0.8)^k - 0.231(0.5)^k + 1.111\right)\varepsilon[k]$$

Z-Transformation

Aufgabe 5:

Z-Transformation:

$$H(z) = \frac{z}{z-0.9} + \frac{z}{z-0.8}$$

$$Y(z) = \frac{0.1z^2}{(z-0.9)(z-0.8)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{5}{z-1} + \frac{4}{z-0.8} - \frac{9}{z-0.9}$$

$$Y(z) = \frac{5z}{z-1} + \frac{4z}{z-0.8} - \frac{9z}{z-0.9}$$

$$y[k] = \left(5 + 4(0.8)^k - 9(0.9)^k\right)$$

Faltung:

$$y[k] = \sum_{m=0}^{k} 0.9^m - \sum_{m=0}^{k} 0.8^m = \frac{1-0.9^{k+1}}{1-0.9} - \frac{1-0.8^{k+1}}{1-0.8}$$

$$= 5 - 9(0.9)^k + 4(0.8)^k$$

Z-Transformation

Aufgabe 6:

Nullstellen:

$$z_1 = j$$

$$z_2 = -j$$

Polstellen:

$$z_1 = 0.707$$

$$z_2 = -0.707$$

$$\frac{H(z)}{z} = \frac{-2}{z} + \frac{1.5}{z-0.707} + \frac{1.5}{z+0.707}$$

$$h[k] = -2\delta[k] + 1.5\left(0.707^k + (-0.707)^k\right)\varepsilon[k]$$

Steady State:

$$z_0 = \frac{\pi}{4}$$

$$H(z_0) = 1.259e^{-j1.249}$$

$$y[k] = 3.148\cos\left(k\frac{\pi}{4} - 1.249\right)$$

Z-Transformation